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Instantons on D7 brane probes and AdS/CFT with flavour

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Abstract

Recent work on adding flavour to the generalized AdS/CFT correspondence is reviewed. In particular, we consider instanton configurations on two coincident D7 brane probes. These are matched to the Higgs branch of the dual field theory. In $AdS_5 \times S^5$, the instanton generates a flow of the meson spectrum. For non-supersymmetric gravity backgrounds, the Higgs branch is lifted by a potential, which has non-trivial physical implications. In particular these configurations provide a gravity dual description of Bose-Einstein condensation and of a thermal phase transition. Based on talk given by J. Erdmenger at the RTN Workshop "Constituents, Fundamental Forces and Symmetries of the Universe", Corfu, Greece, 20-26th September 2005.

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1 Introduction

D7 brane probes have proved a versatile tool for including quark fields into the AdS/CFT correspondence. Strings stretching between the D7s and the D3 branes of the original AdS/CFT construction provide $\mathcal{N} = 2$ fundamental hypermultiplets [1]. Karch and Katz [2] proposed that the open string sector on the world-volume of a probe D7 brane is holographically dual to quark–anti-quark bilinears $\bar{\psi}\psi$. There have been many studies using probe D7s in a variety of gravity backgrounds [3, 4]. In this way a number of non-supersymmetric geometries have been shown to induce chiral symmetry breaking [5, 6] (related analyses are [7, 8]), with the symmetry breaking geometrically displayed by the D7 brane's bending to break an explicit symmetry of the space. Meson spectra are also calculable [9].

In scenarios involving two or more D7 probes, the Higgs branch spanned by squark vevs $\langle \bar{q}q \rangle$ can be identified with instanton configurations on the D7 world-volume [10, 11]. These configurations are the standard four-dimensional instanton solutions living in the four directions of the D7 world-volume transverse to the D3 branes. The scalar Higgs vev in the field theory is identified with the instanton size on the supergravity side. In the case of a probe in AdS space, there is a moduli space for the magnitude of the instanton size or the scalar vev. In [11], the meson spectrum associated with a particular fluctuation about the instanton background is calculated. The spectrum exhibits a non-trivial spectral flow.

In less supersymmetric gravity backgrounds, the moduli space is expected to be lifted by a potential. This potential may have either a stable vacuum selecting a particular scalar vev, or a run-away behaviour. In [12], the Higgs branch of the $\mathcal{N} = 4$ gauge theory at finite temperature and density is analyzed. For the finite temperature case we find a stable minimum for the squark vev which undergoes a first order phase transition as a function of the temperature (or equivalently of the quark mass). On the other hand, in the presence of a chemical potential, the squark potential leads to an instability indicating Bose-Einstein condensation.

Moreover the potential obtained from evaluating the D7 probe action on a static instanton configuration may be used to obtain information about some aspects of the stability of brane embeddings into non-supersymmetric gravity backgrounds. For instance the dilaton-flow background of Constable and Myers [13], which has been used to obtain a gravity dual of chiral symmetry breaking in [5], is expected to be unstable. However we show that the scalar quark potential for the brane embedding into this background is well-behaved and drives the vev to zero [14].

2 Higgs branch AdS/CFT dictionary

Consider a probe of two coincident D7 branes in $AdS_5 \times S^5$. This corresponds to two fundamental hypermultiplets in the dual $\mathcal{N}=2$ gauge theory. The metric of $AdS_5 \times S^5$ is given by

$$ds^2 = H^{-1/2}(r)\eta_{\mu\nu}dx^\mu dx^\nu + H^{1/2}(r)(d\vec{y}^2 + d\vec{z}^2)\,, \tag{1}$$

$$H(r) = \frac{L^4}{r^4}, \quad r^2 = \vec{y}^2 + \vec{z}^2, \quad L^4 = 4\pi g_s N_c (\alpha')^2, \quad \vec{y}^2 = \sum_{m=4}^{7} y^m y^m,$$

$$C^{(4)}_{0123} = H^{-1}, \quad \vec{z}^2 = (z^8)^2 + (z^9)^2, \quad e^\phi = e^{\phi_\infty} = g_s.$$

Two D7-branes are embedded into this geometry according to $z^8 = 0$, $z^9 = (2\pi\alpha')m$. This leads to the induced metric

$$ds^2_{\mathrm{D7}} = H^{-1/2}(r)\eta_{\mu\nu}dx^\mu dx^\nu + H^{1/2}(r)d\vec{y}^2, \quad r^2 = y^2 + (2\pi\alpha')^2 m^2, \;\; y^2 \equiv y^m y^m. \tag{2}$$

The parameter m corresponds to the mass of the fundamental hypermultiplets in the dual $\mathcal{N}=2$ theory.

The effective action describing D7-branes in a curved background is

$$S = T_7 \int \sum_r C^{(r)} \wedge \mathrm{tr}\, e^{2\pi\alpha' F} + T_7 \int d^8\xi \sqrt{g}\, \frac{(2\pi\alpha')^2}{2}\, \mathrm{tr}\left(F_{\alpha\beta}F^{\alpha\beta}\right) + \cdots\,, \tag{3}$$

where we have not written terms involving fermions and scalars. This action is the sum of a Wess-Zumino term, a Yang-Mills term, and an infinite number of corrections at higher orders in α' indicated by $\cdots$ in (3). Since we need to consider at least two flavors (two D7's) in order to have a Higgs branch, the DBI action is non-Abelian. The correspondence between instantons and the Higgs branch suggests that the equations of motion should be solved by field strengths which are self-dual with respect to a flat four-dimensional metric. We work to leading order only in the large 't Hooft coupling expansion generated by AdS/CFT duality, which allows one to only consider the leading term in the α' expansion of the action. Constraints on unknown higher order terms arising from the existence of instanton solutions, as well as the exactly known metric on the Higgs branch, are discussed in [10].

At leading order in α', field strengths which are self dual with respect to the flat four-dimensional metric $ds^2 = \sum_{m=4}^{7} dy^m dy^m$ solve the equations of motion, due to a conspiracy between the Wess-Zumino and Yang-Mills term. Inserting the explicit AdS background values (1) for the metric and Ramond-Ramond four-form into the action for D7-branes embedded as given below (1), with non-trivial field strengths only in the directions y^m, gives

$$\begin{aligned} S &= \frac{T_7(2\pi\alpha')^2}{4} \int d^4x\, d^4y\, H(r)^{-1} \left(-\frac{1}{2}\epsilon_{mnrs}F_{mn}F_{rs} + F_{mn}F_{mn}\right) \\ &= \frac{T_7(2\pi\alpha')^2}{2} \int d^4x\, d^4y\, H(r)^{-1} F_-^2\,, \end{aligned} \tag{4}$$

where $F^-_{mn} = \frac{1}{2}(F_{mn} - \frac{1}{2}\epsilon_{mnrs}F_{rs})$. Field strengths $F^-_{mn} = 0$, which are self-dual with respect to the flat metric $dy^m dy^m$, manifestly solve the equations of motion. These solutions correspond to points on the Higgs branch of the dual $\mathcal{N} = 2$ theory. Strictly speaking, these are points on a mixed Coulomb-Higgs branch if $m \neq 0$ (for details see [11]). We emphasize that in order to neglect the back-reaction due to dissolved D3-branes, we are considering a portion of the moduli space for which the instanton number k is fixed in the large N_c limit.

For $m = 0$, the AdS geometry (1) together with the embedding (2), is invariant under $SO(2,4) \times SU(2)_L \times SU(2)_R \times U(1)_R \times SU(2)_f$. The combination $SU(2)_L \times SU(2)_R$ acts as $SO(4)$ rotations of the coordinates y^m. The $SO(2,4)$ factor is the conformal symmetry of the dual gauge theory. The $SU(2)_L$ factor corresponds to a global symmetry of the dual gauge theory, while $SU(2)_R \times U(1)_R$ corresponds to the R symmetries. Finally $SU(2)_f$ is the gauge symmetry of the two coincident D7-branes which, at the AdS boundary, corresponds to the flavor symmetry of the dual gauge theory.

For $m \neq 0$, the symmetry is broken to $SO(1,3) \times SU(2)_L \times SU(2)_R \times SU(2)_f$. This is broken further if there is an instanton background on the D7-branes. We focus on that part of the Higgs branch, or mixed Coulomb-Higgs branch, which is dual to a single instanton centered at the origin $y^m = 0$. The instanton, in “singular gauge,” is given by

$$A_\mu = 0, \qquad A_m = \frac{2Q^2 \bar{\sigma}_{nm} y_n}{y^2(y^2 + Q^2)}\,, \tag{5}$$

where Q is the instanton size and y_m denote the four coordinate directions parallel to the D7 branes but perpendicular to the D3 branes. Moreover $\bar{\sigma}_{mn} \equiv \frac{1}{4}(\bar{\sigma}_m \sigma_n - \bar{\sigma}_n \sigma_m)$, $\sigma_{mn} \equiv \frac{1}{4}(\sigma_m \bar{\sigma}_n - \sigma_n \bar{\sigma}_m)$, $\sigma_m \equiv (i\vec{\tau}, 1_{2\times 2})$, with $\vec{\tau}$ being the three Pauli-matrices. We choose singular gauge, as opposed to the regular gauge in which $A_n = 2\sigma_{mn} y^m/(y^2+Q^2)$, because of the improved asymptotic behaviour at large y. In the AdS setting, the Higgs branch should correspond to a normalizable deformation of the background at the origin of the moduli space. The singularity of (5) at $y^m = 0$ is not problematic for computations of physical (gauge invariant) quantities. The instanton (5) breaks the symmetries to $SO(1,3) \times SU(2)_L \times \mathrm{diag}(SU(2)_R \times SU(2)_f)$ and corresponds to a point on the Higgs branch $q_{i\alpha} = v\,\varepsilon_{i\alpha}$, $v = \frac{Q}{2\pi\alpha'}$, where $q_{i\alpha}$ are scalar components of the fundamental hypermultiplets, labeled by a $SU(2)_f$ index $i = 1, 2$, and a $SU(2)_R$ index $\alpha = 1, 2$. All the broken symmetries are restored in the ultraviolet (large r), where the theory becomes conformal.

3 Fluctuations and Spectral Flow

The simplest (non-Abelian) ansatz for fluctuations $\mathcal{A}_\mu$ about the instanton background is given by

$$\mathcal{A}_\mu{}^{(a)} = \xi_\mu(k) f(y) e^{ik_\mu x_\mu} \tau^a \,, \quad y^2 \equiv y^m y^m \,. \tag{6}$$

Greek indices lable the four Minkowski directions. (6) is a singlet under $SU(2)_L$ and a triplet under diag$(SU(2)_R \times SU(2)_f)$. The equation of motion for these fluctuations becomes

$$0 = \left[\frac{M^2 R^4}{(y^2 + (2\pi\alpha')^2 m^2)^2} - \frac{8Q^4}{y^2(\rho^2 + Q^2)^2} + \frac{1}{y^3}\partial_y(y^3 \partial_y) \right] f(y) \,, \tag{7}$$

where $M^2 = -k_\mu k_\mu$. To determine the spectrum, we find the values of M^2 for which this equation admits normalizable solutions. The spectrum is plotted in Figure 1.

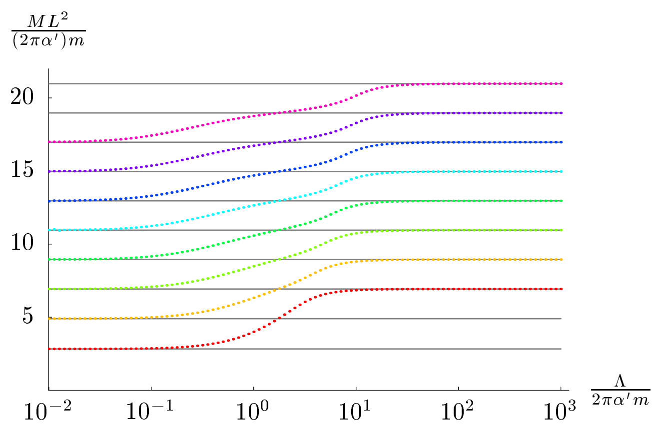

Figure 1. Meson masses as function of the Higgs VEV.

When flowing from zero to infinite instanton size, the meson spectrum $M(n, l)$ is shifted to a higher spherical harmonic on S^3 with $l = 2$. It is straightforward to explain this shift. In the limits of zero or infinite instanton size, we have

$$0 = \left[\frac{\tilde{M}^2}{(\tilde{y}^2 + 1)^2} - \frac{l(l+2)}{\tilde{y}^2} + \frac{1}{\tilde{y}^3}\partial_{\tilde{y}}(\tilde{y}^3 \partial_{\tilde{y}}) \right] f(\tilde{y}), \quad \tilde{y} \equiv \frac{y}{2\pi\alpha' m} \,, \tag{8}$$

with $l = 0, 2$. This is a special case of the equations found in [3] for fluctuations about the trivial background without any instantons of the form

$$\mathcal{A}^\mu = \xi^\mu(k) e^{ik_\mu x^\mu} f(y) \mathcal{Y}_l(S^3) \,. \tag{9}$$

Here $\mathcal{Y}_l$ are spherical harmonics on S^3 corresponding to the $(l/2, l/2)$ representation of $SU(2)_L \times SU(2)_R$. In [3] it was found that the spectrum is given by

$$\tilde{M}^2 = 4(n + l + 1)(n + l + 2) \,. \tag{10}$$

In our case, at infinite instanton size in singular gauge, the instanton is given by

$$A_n = 2\frac{\bar{\sigma}_{mn} y^m}{y^2} \,. \tag{11}$$

This instanton may be removed by a gauge transformation of the form $U = \sigma^m y^m/|y|$ which gives $A_n = 0$. In this gauge the fluctuations become

$$\mathcal{A}_\mu{}^{(a)} = \xi_\mu(k) f(y) e^{ik_\mu x_\mu} \frac{y^m y^n}{y^2} \sigma^m \tau^a \bar{\sigma}^n \,. \tag{12}$$

Here $\sigma^m \tau^a \bar{\sigma}^n$ corresponds exactly to the $l = 2$ spherical harmonic. This explains the shift in spectrum.

4 Higgs potential for non-supersymmetric backgrounds

4.1 Chemical potential

As the simplest example of a potential generated on the Higgs branch, we first consider [12] the case of finite chemical potential and zero temperature. We consider a nonzero chemical potential for the isospin. We allow a spurious gauge field associated with the τ^3 component of isospin to acquire a VEV, μ, in its A^0 component. This includes generic fermion and scalar Lagrangian terms for fields with isospin charge e of the form

$$\delta\mathcal{L} = -\mu e \bar{\psi} \tau^3 \gamma^0 \psi + \mu^2 e^2 |\phi|^2 \,. \tag{13}$$

The first term is a source for the fermionic isospin number density. In the path integral, this term places the theory at finite density. The second term is an unbounded scalar potential which renders the theory unstable, such that Bose-Einstein condensation is expected.

This is described in the dual gravity picture as follows. We add a background A^0 in the fixed instanton background,

$$A^0 = \begin{pmatrix} \mu & 0 \\ 0 & -\mu \end{pmatrix}, \qquad A_n = A_n^{\text{instanton}} \,. \tag{14}$$

On the slice of the Higgs branch corresponding to the single instanton configurations (5) with modulus Q, the effective potential at quadratic order in μ can be determined by inserting (14) into the D7-brane action. Since the instanton configuration is static we have $\int d^4x \; V(Q) = -S_{D7}$, which gives

$$\begin{aligned} V(Q) = T_7 \frac{(2\pi\alpha')^2}{g_s} \int d^4 y \, \mathrm{tr} \Bigg(& \frac{1}{2} \frac{(y^2+m^2)^2}{R^4} F^-_{mn} F^-_{mn} + 2 F_{m\mu} F_{m\nu} \eta^{\mu\nu} \\ & + \frac{R^4}{(y^2+m^2)^2} F_{\mu\nu} F_{\alpha\beta} \eta^{\mu\alpha} \eta^{\nu\beta} \Bigg) \,, \end{aligned} \tag{15}$$

with $y^2 = y^m y^m$. We have split the action into the pieces involving F in the x and y directions, indicated by Greek and Roman indices respectively, as well as mixed terms. For the background (14), the only non-zero contribution to the potential comes from the mixed term $\mathrm{tr}\, F_{\mu m} F_{\nu m} \eta^{\mu\nu} = -\mathrm{tr}\, [A_0, A_n]^2$, giving

$$V(Q) = -T_7 \frac{2(4\pi\alpha')^2}{g_s} \mu^2 \int d^4y \frac{Q^4}{y^2(y^2+Q^2)^2} = -T_7 \frac{2(4\pi^2\alpha')^2}{g_s} \mu^2 Q^2 \,. \tag{16}$$

This potential displays an instability which may be interpreted as Bose-Einstein condensation in the dual field theory.

4.2 Thermal phase transition

The gravitational dual of $\mathcal{N} = 4$ gauge theory at large 't Hooft coupling and finite temperature is given by the AdS-Schwarzschild black-hole background [15]. The latter belongs to a general class of supergravity solutions which, in a choice of coordinates convenient for our purposes, have the form

$$\begin{aligned}
& ds^2 = f(r)(d\vec{x}^2 + g(r) d\tau^2) + h(r)(\sum_{m=4}^{7} dy^m dy^m + \sum_{i=8}^{9} dZ^i dZ^i), \\
& e^{-\Phi} = \phi(r), \qquad r^2 = y^m y^m + Z^i Z^i \,, \\
& F^{(5)} = 4R^4 (V_{S^5} +^* V_{S^5}) = dC_{(4)}, \qquad C_{(4)}|_{0123} = s(r)\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \,,
\end{aligned} \tag{17}$$

For the AdS-Schwarzschild solution, we have

$$f(r) = \frac{4r^4 + b^4}{4r^2 R^2}, \quad g(r) = \left(\frac{4r^4 - b^4}{4r^4 + b^4}\right)^2, \quad h(r) = \frac{R^2}{r^2}, \quad s(r) = \frac{r^4}{R^4}\left(1 + \frac{b^8}{16r^8}\right) \,. \tag{18}$$

The coordinates $\vec{x}$ are the spatial coordinates of the dual gauge theory and τ is the Euclidean time direction, which is compactified on a circle of radius b^{-1}, corresponding to the inverse temperature. Note that the temperature $T \sim b$ only enters to the fourth power. The D7 embedding in this background is given by $Z^9 = 0$, $Z^8 = z(y)$.

The potential generated on the Higgs branch was calculated in [12]. Specifically, the action is evaluated on the space of field strengths which are self-dual[1] with respect to the induced metric in the directions transverse to $\tau, \vec{x}$;

$$V = \frac{T_7 (2\pi\alpha')^2}{2} \left(\frac{1}{g_s} \int d^4y\, C^{(4)}_{0123}\, \epsilon_{mnrs}\, \mathrm{tr}\, F_{mn} F_{rs} - \frac{1}{2} \int d^4y \sqrt{-\mathrm{det} G}\, \mathrm{tr}\, F^{mn} F_{mn} \right), \tag{19}$$

[1]There are couplings between world-volume scalars and field strengths at higher orders in α' which could alter the embedding. However we only consider the leading term in a large 't Hooft coupling expansion for which these couplings can be neglected.

where F_{mn} is self-dual with respect to the metric $ds_\perp^2 = h(r)\left((1+z'(y)^2)dy^2 + y^2 d\Omega_3^2\right)$. This metric is conformally flat. With new coordinates $\tilde{y}(y)$ such that $ds^2 = \alpha(\tilde{y})(d\tilde{y}^2 + \tilde{y}^2 d\Omega_3^2)$, the instanton configurations (self-dual field strengths) take the usual form.

To compute $V(Q)$ in general requires knowledge of the embedding function $z(y)$, which has been computed by a numerical shooting technique in [5]. Imposing boundary conditions for the large y behaviour, and requirig smooth behaviour in the interior, such that an RG flow interpretation is possible, leads to a dependence of the chiral quark condensate $\langle\bar{\psi}\psi\rangle$ on the quark mass m and on the temperature. Depending on the ratio m/b, there are two types of solutions, which differ by the topology of the D7-branes. At large r (or y) the geometry of the D7-branes is $AdS_5 \times S^3$ and the topology of the $r \to \infty$ boundary is $S^1 \times R^3 \times S^3$. For sufficiently large m/b, the S^3 component of the D7-geometry contracts to zero size at finite $r > b$. In this case the D7-brane "ends" before reaching the horizon at $r = b$. However, for sufficiently small m/b, the D7-brane ends at the horizon, at which point the thermal S^1 contracts to zero size. Both these types of solutions are plotted in figure 2. There is a first order phase transition at the critical value of $m/b \approx 0.92$ where the two types of solution meet [5, 16]. The $\langle\bar{\psi}\psi\rangle$ condensate is non-zero on both sides of this transition, although there is a discontinuous jump in its value.

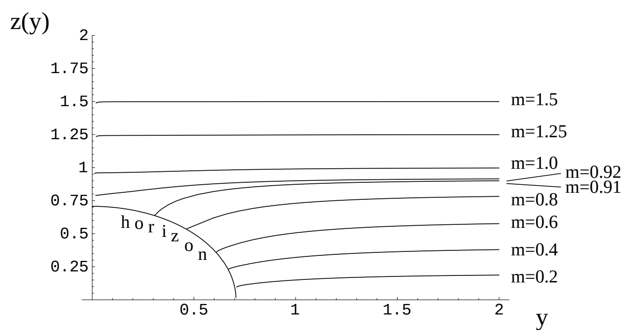

Figure 2. Brane embeddings in AdS-Schwarzschild for different values of the quark mass (with $b = 1$).

This same phase transition is also observed in the Higgs potential as shown in Figure 3. For $m = \infty$, the potential is flat as in the AdS case. For smaller and smaller values of m, a minimum forms at $Q = 0$, until at a critical value of m, the minimum of the potential moves to a finite value of Q.

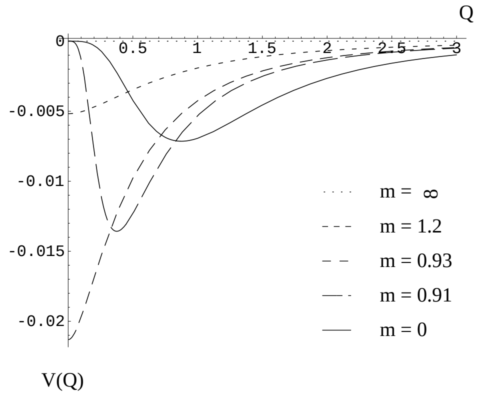

Figure 3. Potential $V(Q)$ as a function of the instanton size / Higgs VEV Q for various values of the quark mass m.

In Figure 4 the Higgs vev Q_0, for which the Higgs potential is minimised, is plotted versus the quark mass. This clearly displays the first order nature of the phase transition. Q_0 is a suitable order parameter for this transition.

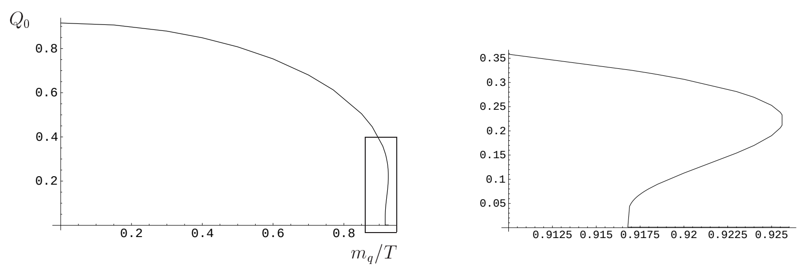

Figure 4. Position of the minimum of the potential Q_0 versus the bare quark mass m, zoom of the critical region.

Moreover we have also calculated [14] the Higgs potential for the background of Constable and Myers (CM) [13] (see also [17]). This dilaton-flow geometry is asymptotically AdS at large radius, but is deformed in the interior of the space by an R-chargeless parameter of dimension four (b^4 in what follows). It is interpreted as being dual to $\mathcal{N} = 4$ gauge theory with a non-zero expectation value for $\mathrm{tr}\, F^2$. It was used in [5] to study chiral symmetry breaking because of its particularly simple form with a flat six-dimensional plane transverse to the D3 branes. The core of the geometry is singular[2]. In Einstein frame, the Constable-Myers geometry is given by

$$ds^2 = H^{-1/2} K^{\delta/4} dx_4^2 + H^{1/2} K^{(2-\delta)/4} \frac{u^4 - b^4}{u^4} \sum_{i=1}^{6} du_i^2, \tag{20}$$

where

$$K = \left(\frac{u^4 + b^4}{u^4 - b^4} \right), \qquad H = K^{\delta} - 1, \qquad \delta = \frac{R^4}{2b^4}, \qquad \Delta^2 = 10 - \delta^2 \,,$$

$$e^{2\Phi} = g_s^2 K^{\Delta}, \qquad C_{(4)} = (g_s\, H)^{-1}\, dt \wedge dx \wedge dy \wedge dz. \tag{21}$$

[2]This singularity may presumably be lifted by the D3 branes forming some sort of fuzzy sphere in the interior of the space.

In this geometry, D7 brane probes are repelled by the central singularity, giving rise to chiral symmetry breaking [5]. This is shown in Figure 5a), where $\sum_{i=1}^{4} u_i^2 = \rho^2$.

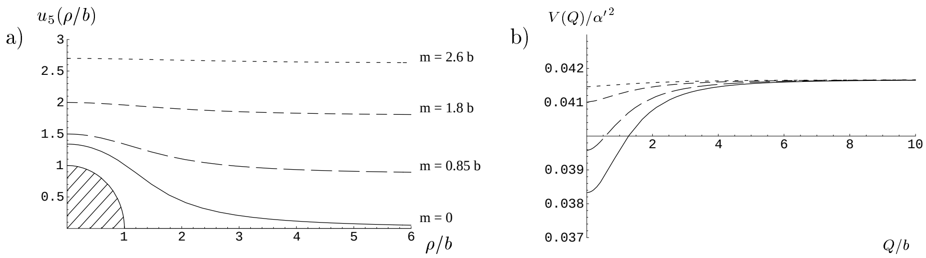

Figure 5. a) Plot of the D7 brane embedding in the Constable-Myers geometry for different values of the quark mass m. b) Potential versus the Higgs vev Q for the values of m shown in 5a).

The Higgs potential (19) for the Constable-Myers background is displayed in Figure 5b). We see that although the Constable-Myers background is expected to be unstable, the Higgs potential has a stable minimum at $Q = 0$. Thus it is well-behaved and drives the vev to zero. The minimum of the Higgs potential becomes more and more pronounced when the the quark mass is sent to zero and the brane is strongly bent, as required for spontaneous chiral symmetry breaking. For large Q the potential has the form of a constant minus a $1/Q^4$ term (remember $Q^2 = \langle \bar{q} q \rangle$ with q the scalar quarks). This behaviour is determined essentially by dimensional counting since the supersymmetry breaking operator, $\mathrm{tr} F^2$, is dimension four.

Let us provide some intuition for why the brane configuration disfavours large instantons. We suggest the essential reason is that the background metric causes volume elements to expand for small ρ: The D7 brane bends away from the singularity in order to minimize its world-volume. We expect that the instanton action will grow with the size of the instanton in the region where the brane is strongly bent, preferring zero size instantons. This argument does not apply in pure AdS because the four-form term conspires to cancel the $\sqrt{\det G}$ volume term. However when supersymmetry is broken, this cancellation no longer works, and the increase in the volume term is the stronger effect. This ensures a stable minimum for the Higgs vev.

Finally, we have also shown [14] that embedding D7 brane probes into the Yang-Mills* background [18] leads to a Higgs potential which is bounded.

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